

# 2016 Statistics Advanced Higher Finalised Marking Instructions

## © Scottish Qualifications Authority 2016

The information in this publication may be reproduced to support SQA qualifications only on a non-commercial basis. If it is to be used for any other purposes written permission must be obtained from SQA's NQ Assessment team.

Where the publication includes materials from sources other than SQA (secondary copyright), this material should only be reproduced for the purposes of examination or assessment. If it needs to be reproduced for any other purpose it is the centre's responsibility to obtain the necessary copyright clearance. SQA's NQ Assessment team may be able to direct you to the secondary sources.

These Marking Instructions have been prepared by Examination Teams for use by SQA Appointed Markers when marking External Course Assessments. This publication must not be reproduced for commercial or trade purposes.



#### **General Marking Principles for Advanced Higher Statistics**

This information is provided to help you understand the general principles you must apply when marking candidate responses to questions in this Paper. These principles must be read in conjunction with the detailed marking instructions, which identify the key features required in candidate responses.

For each question the marking instructions are generally in two sections, namely Illustrative Scheme and Generic Scheme. The Illustrative Scheme covers methods which are commonly seen throughout the marking. The Generic Scheme indicates the rationale for which each mark is awarded. In general, markers should use the Illustrative Scheme and only use the Generic Scheme where a candidate has used a method not covered in the Illustrative Scheme.

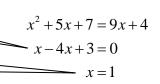
- (a) Marks for each candidate response must <u>always</u> be assigned in line with these General Marking Principles and the Detailed Marking Instructions for this assessment.
- (b) Marking should always be positive. This means that, for each candidate response, marks are accumulated for the demonstration of relevant skills, knowledge and understanding: they are not deducted from a maximum on the basis of errors or omissions.
- (c) If a specific candidate response does not seem to be covered by either the principles or detailed Marking Instructions, and you are uncertain how to assess it, you must seek guidance from your Team Leader.
- (d) Credit must be assigned in accordance with the specific assessment guidelines.
- (e) One mark is available for each •. There are no half marks.
- (f) Working subsequent to an error must be **followed through**, with possible credit for the subsequent working, provided that the level of difficulty involved is approximately similar. Where, subsequent to an error, the working for a follow through mark has been eased, the follow through mark cannot be awarded.
- (g) As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Unless specifically mentioned in the marking instructions, a correct answer with no working receives no credit.
- (h) Candidates may use any mathematically correct method to answer questions except in cases where a particular method is specified or excluded.
- (i) As a consequence of an error perceived to be trivial, casual or insignificant, eg  $6 \times 6 = 12$  candidates lose the opportunity of gaining a mark. However, note the second example in comment (j).

Where a transcription error (paper to script or within script) occurs, the candidate (j) should normally lose the opportunity to be awarded the next process mark, eg

This is a transcription error and so the mark is not awarded.

Eased as no longer a solution of a quadratic equation so mark is not awarded.

Exceptionally this error is not treated as a transcription error as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.



$$x^2 + 5x + 7 = 9x + 4$$

$$x-4x+3=0$$

$$(x-3)(x-1)=0$$

$$r=1$$
 or  $r=1$ 

$$x = 1$$
 or 3

Horizontal/vertical (k)

marking

Where a question results in two pairs of solutions, this technique should be applied, but only if indicated in the detailed marking instructions for the question.

Example:

• 
$$x = 2$$
  $x = -4$ 
•  $y = 5$   $y = -7$ 

Horizontal: • $^5$  x = 2 and x = -4 Vertical: • $^5$  x = 2 and y = 5 • $^6$  y = 5 and y = -7•  $^{6}$  y = 5 and y = -7

Markers should choose whichever method benefits the candidate, but **not** a combination of both.

**(l)** In final answers, unless specifically mentioned in the detailed marking instructions, numerical values should be simplified as far as possible, eg:

 $\frac{15}{12}$  must be simplified to  $\frac{5}{4}$  or  $1\frac{1}{4}$   $\frac{43}{1}$  must be simplified to 43

 $\frac{15}{0.3}$  must be simplified to 50  $\frac{\frac{4}{5}}{3}$  must be simplified to  $\frac{4}{15}$ 

 $\sqrt{64}$  must be simplified to 8\*

- \*The square root of perfect squares up to and including 100 must be known.
- (m) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

- (n) Unless specifically mentioned in the marking instructions, the following should not be penalised:
  - Working subsequent to a correct answer
  - Correct working in the wrong part of a question
  - Legitimate variations in numerical answers/algebraic expressions, eg angles in degrees rounded to nearest degree
  - Omission of units
  - Bad form (bad form only becomes bad form if subsequent working is correct), eg  $(x^3 + 2x^2 + 3x + 2)(2x + 1)$  written as  $(x^3 + 2x^2 + 3x + 2) \times 2x + 1$

$$2x^4 + 4x^3 + 6x^2 + 4x + x^3 + 2x^2 + 3x + 2$$
 written as  $2x^4 + 5x^3 + 8x^2 + 7x + 2$  gains full credit

- Repeated error within a question, but not between questions or papers
- (o) In any 'Show that...' question, where the candidate has to arrive at a required result, the last mark of that part is not available as a follow-through from a previous error unless specified in the detailed marking instructions.
- (p) All working should be carefully checked, even where a fundamental misunderstanding is apparent early in the candidate's response. Marks may still be available later in the question so reference must be made continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that the candidate has gained all the available marks.
- (q) Scored-out working which has not been replaced should be marked where still legible. However, if the scored out working has been replaced, only the work which has not been scored out should be marked.
- (r) Where a candidate has made multiple attempts using the same strategy and not identified their final answer, mark all attempts and award the lowest mark.

#### For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

# Detailed Marking Instructions for each question

Ç	(uestion	Generic Scheme	Illustrative Scheme	Max Mark
1	(a)	• 1 appropriate diagram	• ¹ dotplot, stem-and-leaf or boxplot	1
Not	es:	,		
Cor	nmonly	Observed Responses:		
	(b)	• <sup>2</sup> correct quartiles	• $^{2}$ 16·5 and 27·5	3
		• <sup>3</sup> calculate upper fence	• $^{3}$ 27 · 5 + 1 · 5 × 11 = 44	
		• 4 appropriate comment	• 4 48 is an outlier	
Not	es:			
Cor	nmonly	Observed Responses:		
2	(a)	• <sup>1</sup> appropriate comment	• 1 eg surgeons may not be sampled in the same proportions as operations performed	1
		r valid comment is that some surgeo  Observed Responses:	ons may not be represented at all	
	(b)	• <sup>2</sup> appropriate strategy	• <sup>2</sup> determine the proportion of operations performed by each surgeon and sample the 100 patients in the same proportion	2
		• 3 correct sampling method	• <sup>3</sup> stratified sampling	
Not	es:	·		
Cor	nmonly	Observed Responses:		
	(c)	• <sup>4</sup> appropriate comment	• 4 eg waiting time for the operation, recovery time after the operation, quality of hospital care	1
	es: other a	ppropriate comments are acceptable	e	

Qı	Question		Generic Scheme	Illustrative Scheme	Max Mark
3	(a)		•¹ correct test statistic	$\bullet^1  t = \frac{\overline{x} - \mu}{s / \sqrt{n}}$	4
			• $^2$ correct value of $t$	$\bullet^2 = \frac{147 \cdot 8 - 150}{2 \cdot 379 / \sqrt{12}} = -3 \cdot 203$	
			• <sup>3</sup> appropriate critical value and inequality	• 3 1% cv is $t_{11,0.99} = -2.718 > -3.203$	
			• <sup>4</sup> appropriate conclusion	<ul> <li>furnishing strong evidence that the population mean content weight is less than 150 g</li> </ul>	

## Notes:

- 1. a formal test is not required and so neither are formal hypotheses
- 2. the alternative (and acceptable) *p*-value approach (hereafter the PvA) would record that  $P(t_{11} < -3.203) = 0.0042 < 0.01$  etc
- 3. constructing an appropriate confidence interval would also be acceptable

# **Commonly Observed Responses:**

	(b)	• <sup>5</sup> appropriate comment	• <sup>5</sup> the population variance is	1
			unknown	

## Notes:

## **Commonly Observed Responses:**

4	(a)	(i)	• ¹ appropriate comment	•¹ distribution is skewed	1
		(ii)	• <sup>2</sup> correct statistics	$\bullet^2 \stackrel{-}{x} = 7 \cdot 1$ and $s = 2 \cdot 43$	1

## Notes:

# **Commonly Observed Responses:**

(b)	• <sup>3</sup> appropriate strategy	$\bullet^3 \overline{X} \approx N$	4
	• <sup>4</sup> calculate parameters	$\bullet^4 \left(7 \cdot 1, \frac{5 \cdot 893}{20}\right)$	
	• <sup>5</sup> calculate z	$\bullet^5 z = \frac{8 \cdot 1 - 7 \cdot 1}{\sqrt{\frac{5 \cdot 893}{20}}} = 1 \cdot 84$	
	• <sup>6</sup> calculate probability	$\bullet^6 p = 0.0329$	

## Notes:

1. it is a requirement to record that normality is approximate

Question		ion	Generic Scheme	Illustrative Scheme	Max Marks
5	(a)	(i)	• <sup>1</sup> appropriate strategy	$\bullet^1  \mu = \overline{x} \pm t_{\nu, 1 - \alpha/2} \frac{s}{\sqrt{n}}$	5
			$ullet^2$ correct $\overline{x}$ and $s$	$\bullet^2 \ \overline{x} = 4.9 \ \text{and} \ s = 0.462$	
			•³ correct t	$\bullet^3 t_{8, 0.95} = 1.86$	
			• <sup>4</sup> calculate interval	•4 (4.61, 5.19)	
			• <sup>5</sup> appropriate assumption	• <sup>5</sup> the yield of tomatoes is normally distributed	
1.			lid assumption is that the plants are	chosen at random	
Coı	mmor	ily Ob	served Responses:		
		(ii)	• <sup>6</sup> interpret <i>t</i> interval	● 6 4·75 lies within calculated interval	2
			• <sup>7</sup> appropriate comment	• 7 so no evidence, at the 10% level of significance, that the mean yield has changed, by watering plants once a week	
No	tes:				
Coı	mmor	ıly Ob	served Responses:		
	(b)		• 8 appropriate comment	• 8 eg temperature, soil type, application of fertiliser, hours of sunshine	1
Not	tes:				
Coi	mmor	ıly Ob	served Responses:		

Question		Generic Scheme	Illustrative Scheme	Max Mark
6	(a)	• <sup>1</sup> identify correct WECO rule	• 1 8 consecutive points lie above the centre line	1
Note 1. re		nce to a WECO rule should be made		
Com	monly	Observed Responses:		
	(b)	•² use binomial distribution	$\bullet^2 X \sim \mathbf{B}\left(14, \frac{1}{2}\right)$	3
		• <sup>3</sup> appropriate strategy	$\bullet^3 2 P(X = 12)$	
		• <sup>4</sup> calculate probability	• 4 0.0111	
Note	es:			
Com	monly	Observed Responses:		
	(c)	• <sup>5</sup> correct months	• <sup>5</sup> 14 and 15	1
Note	es:		L	
Com	monly	Observed Responses:		
	(d)	• 6,7 appropriate strategy		3
		• 8 appropriate simplification	•8 $n = \frac{9 \times 0.025 \times 0.975}{0.015^2} = 975$	
Note	es:			
Com	monly	Observed Responses:		
	(e)	• 9 appropriate values	• $^{9}$ $np = 24.375$ and $nq = 950.625$	4
		• <sup>10</sup> appropriate comment	since both $np$ and $nq > 5$ it might be appropriate to use a normal approximation to the binomial distribution	
		• 11 correct strategy	$\bullet^{11} p(Z>1) =$	
		• <sup>12</sup> calculate percentage	• <sup>12</sup> 16%	
Note	es:		·	

Question		on	Generic Scheme	Illustrative Scheme	
7	(a)		•¹ correct probability	•1 1/3	1
Not	es:		<u> </u>		
Cor	nmor	nly C	Observed Responses:		
	(b)		• <sup>2</sup> correct probabilities	$\bullet^2 \frac{50}{150} \times \frac{49}{149} \times \frac{48}{148}$	2
			• <sup>3</sup> calculate probability	•³ 0·0356	
Not	es:				•
Cor	nmor	nly C	Observed Responses:		
	(c)	(i)	• 4 appropriate strategy	• 4 eg create tree diagram	4
			• <sup>5</sup> continue strategy	• <sup>5</sup> annotate clearly	
				1/ <sub>3</sub> V 0·94	-
			• <sup>6</sup> complete strategy	$\bullet^6 \frac{1}{3} \cdot 0.94 + \frac{2}{3} \cdot 0.07 =$	

# Notes:

# **Commonly Observed Responses:**

• <sup>7</sup> calculate probability

	(ii)	• 8 appropriate strategy	•8 $P(V   +) = \frac{P(V \cap +)}{P(+)}$	3
		• 9 substitute probabilities	$\bullet^9 = \frac{0.94/3}{0.36}$	
		• <sup>10</sup> calculate probability	$\bullet^{10} = 0.8704$	

 $\bullet^7 = 0.36$ 

# Notes:

Q	uesti	on	Generic Scheme	Illustrative Scheme	Max Mark
8	(a)		• 1 appropriate hypotheses	$ \mathbf{H}_{o}: \mathbf{p}_{d} = \mathbf{p}_{c}  \mathbf{H}_{1}: \mathbf{p}_{d} > \mathbf{p}_{c} $	7
			• <sup>2</sup> correct test statistic	$\bullet^2 Z = \frac{p_d - p_c}{\sqrt{pq\left(\frac{1}{n_d} + \frac{1}{n_c}\right)}}$	
			• ³ correct p	$\bullet^3 \ p = \frac{75 + 65}{200} = 0.7$	
			• <sup>4</sup> calculate z	$\bullet^4 z = \frac{0.75 - 0.65}{\sqrt{0.7.0.3 \left(\frac{1}{100} + \frac{1}{100}\right)}} = 1.54$	
			• <sup>5</sup> correct critical value	• <sup>5</sup> the 5% cv is 1·64	
			• <sup>6</sup> deal with H₀	• 6 1.54 < 1.64 so we cannot reject Ho at the 5% level of significance	
			• <sup>7</sup> appropriate conclusion	• <sup>7</sup> and conclude that there is no evidence that the new drug is effective	
	t <b>es:</b> the f	PvA v	would record that $P(Z > 1.54) = 0.0614$	4 > 0·05 etc	
Cor	nmoi	nly C	Observed Responses:		
	(b)		• 8 appropriate comment	• 8 the new drug may have reduced the recovery rate	1
Not	tes:	•			
Cor	nmoi	nly C	Observed Responses:		

Question		Generic Scheme	Illustrative Scheme	Max Marks
9	(a)	• <sup>1</sup> appropriate assumption	• 1 the sporting injuries are all random/independent of each other.	2
		• <sup>2</sup> appropriate assumption	• <sup>2</sup> the injuries occur at a constant mean rate of 4 per week.	
Not	es:			
Cor	nmonl	y Observed Responses:		
	(b)	• <sup>3</sup> correct standard deviation	$\bullet^3 \sqrt{4} = 2$	3
		• 4 appropriate probability	• $^{4}$ P(X > 8) =	
		• <sup>5</sup> calculate probability	• <sup>5</sup> 0·0214	
Not	es:	•		
Cor	nmonl	y Observed Responses:		
	(c)	• <sup>6</sup> correct strategy	• <sup>6</sup> Y~Po(152)	5
		• <sup>7</sup> correct approximation	$\bullet^7 Y \approx N(152,152)$	
		• 8 correct strategy	• 8 & 9 $P(Y < 140) = P\left(Z < \frac{139 \cdot 5 - 152}{\sqrt{152}}\right)$	
		• 9 continuity correction		
		• 10 calculate probability	• <sup>10</sup> 0·1562	
Not	es:			
Cor	nmonl	y Observed Responses:		
	(d)	• 11 appropriate comment	• 11 team sports may result in injuries which are not independent.	1
1.		er valid comment would record the fa hout the year	act that most contact sports do not take pl	lace

Question		on	Generic Scheme	Illustrative Scheme	Max Mark
10	(a)		• 1 appropriate strategy	$\bullet^1 1p + 2(1-p) = \frac{7}{4}$	3
			• <sup>2</sup> correct probability	$\bullet^2  p = \frac{1}{4}$	
			• ³ tabulate pd	$ \begin{array}{c cccc}  & x & 1 & 2 \\ \hline  & p(x) & 1/4 & 3/4 \end{array} $	
Note	es:				
Com	ımon	ly Ob	served Responses:		
	(b)		• <sup>4</sup> correct E(Y)	$\bullet^4 \frac{8}{5} (1 \cdot 6)$	2
			$ullet^5$ calculate $\mathrm{V}(Y)$		
Note	es:				
Com	mon	ly Ob	served Responses:		
	(c)		• <sup>6</sup> calculate E(Z)	$\bullet^6 \frac{73}{20} (3.65)$	3
			• <sup>7</sup> correct strategy	$\bullet^7 3^2 V(X) + V(Y)$	
			• 8 calculate V(Z)	$\bullet^8 = \frac{771}{400} \left( 1.9275 \right)$	
Note	es:	1		-	
Com	mon	lv Oh	served Responses:		

Question		on	Generic Scheme	Illustrative Scheme	Max Marks
11	(a)		• <sup>1</sup> appropriate hypotheses	• $H_0$ : $\mu_d = 0$ $H_1$ : $\mu_d > 0$	7
			•² correct statistics	• $\bar{x}_d = 1.175$ and $s_d = 4.272$	
			•³ correct test statistic	$\bullet^3  t_{n-1} = \frac{\overline{x}_d \sqrt{n}}{s_d}$	
			• <sup>4</sup> calculate <i>t</i>	$\bullet^4 = \frac{1 \cdot 175\sqrt{12}}{4 \cdot 272} = 0.9528$	
			• <sup>5</sup> correct critical value	• 5% cv is $t_{11,0.950} = 1.796$	
			• <sup>6</sup> deal with H <sub>0</sub>	• $0.9528 < 1.796$ so we cannot reject H $_0$ at the 5% significance level	
			• <sup>7</sup> appropriate conclusion	• <sup>7</sup> and conclude that there is no evidence to support Darwin	

## Notes:

- 1. the PvA would record that  $P(t_{11} > 0.9528) = 0.1806 > 0.05$  etc
- 2. a maximum of 5 marks is available for performing the wrong test and the use of a non-parametric test or a test for independent samples receives little credit

# **Commonly Observed Responses:**

(b)	• appropriate test	• 8 Wilcoxon signed-rank test	6
	• 9 appropriate assumption	• 9 assumes that the distribution of height differences is symmetrical	
	• <sup>10</sup> deal with the zero difference	• 10 discard the zero difference	
	• <sup>11</sup> correct statistics	• $^{11}$ $n = 11$ , $W_{-} = 20$ , $W_{+} = 46$	
	• 12 correct critical value	• <sup>12</sup> 5% cv is 13	
	• <sup>13</sup> appropriate conclusion	• 13 same conclusion - no evidence to support Darwin	

#### Notes:

- 1. for  $\bullet^{11}$  the only requirement is  $W_{-}=20$
- 2. the use of a parametric test or a test for independent samples receives little credit

## **Commonly Observed Responses:**

	(c)	• <sup>14</sup> appropriate comment	• 14 the pairing of data helps to reduce the effect of other variables, apart from method of pollination	1
--	-----	-------------------------------------	---	---

## Notes:

Question	Generic Scheme	Illustrative Scheme	Max Marks
12 (a)	• <sup>1</sup> appropriate comment	• 1 there appears to be a weak linear association between attractiveness and performance	1
Notes:			
Commonly (	Observed Responses:		
(b)	• <sup>2</sup> correct strategy	$\bullet^2 r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$	4
	$\bullet$ orrect $r$	$\bullet^3 = 0.4001$	
	• <sup>4</sup> appropriate comment	• 4 measuring the strength of the linear association between $x$ and $y$	
	• <sup>5</sup> appropriate comment	• 5 the association is weak	
Notes:			
Commonly (	Observed Responses:		
(c)	• <sup>6</sup> correct <i>t</i>	$\bullet^6 \ t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}} = 3 \cdot 6$	2
	• <sup>7</sup> appropriate explanation	• 7 5% cv is $t_{68,0.975} = 1.96 < 3.6$ so	
		there is evidence that the pmcc is significantly different from 0	
Notes:			
the PvA	would record that $2P(t_{68} > 3.6) = 0.$	0006 < 0.05 etc	
Commonly (	Observed Responses:		
(d)	• 8 appropriate comment	• 8 with a large enough sample even a weak correlation may be significant	2
	• 9 appropriate conclusion	• 9 the association between attractiveness and performance is weak, but statistically significant	
Notes:			
Commonly (	Observed Responses:		
(e)	• 10 appropriate reason	• 10 low correlation means a linear relationship is questionable so a linear regression model would have a poor fit	1
Notes:	valid reason would be concerned w	rith the fact that predictability would be	
	e due to the low pmcc	The fact that predictability would be	